

MATH 2850: SOLVING IVPs WITH UNIT STEP FUNCTIONS

EXAMPLE: Solve: $y' + y = f(t)$; $y(0) = 0$ where $f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 1 \\ -1 & \text{if } t \geq 1 \end{cases}$

Ans: $y(t) = 1 - e^{-t} - 2\mathcal{U}(t - 1)(1 - e^{-(t-1)})$

EXAMPLE: Solve: $y'' + y = f(t)$, $y(0) = 1$, $y'(0) = 2$ where $f(t) = \begin{cases} t & \text{if } 0 \leq t < 2\pi \\ -2t & \text{if } t \geq 2\pi \end{cases}$

Ans: $y(t) = t + \sin(t) + \cos(t) - 3\mathcal{U}(t - 2\pi) (t - \sin(t) - 2\pi \cos(t))$

HOMEWORK: Section 8.5: Pg. 438: 1 - 17 odd